Chapter 1

A Review of Math Basics

In This Chapter

- Taking a look at important sets of numbers
- Looking back at addition, subtraction, multiplication, and division
- Examining operations
- Looking into exponents, roots, and absolute values
- Reviewing factors and multiples

You more than likely

already have some type of background in mathematics.

In this chapter,

we will be discussing basic math ideas.

Topics such as the Basic Four operations (addition, subtraction, multiplication, and division)

will be covered.

We will also get to know different sets of numbers.

The properties and operations

that this chapter goes over

will come in handy when solving problems.

Lastly, I'll bring back memories

with a review of factors and multiples.

My goal is to use this book

to help you delve deeper into the world of math.

I want to see everyone move onward and upward.

Your success is my goal!

Sets of Numbers

We can use the real number line

to deal with different *sets* (or groups)

of real numbers.

It is necessary to be familiar

with the various terms

that are used to clarify different types of real numbers.

Natural Numbers, Counting Numbers, or Positive Integers:

This set begins at 1.

It goes on infinitely.

{1, 2, 3, 4, ...}

Whole Numbers, Nonnegative Integers:

{0, 1, 2, 3, ...}

This set begins at 0.

It goes on forever.

The only difference between this set and the set of natural numbers

is the inclusion of 0.

Negative Integers:

{..., -3, -2, -1}

This is the set of all negative integers.

Nonpositive Integers:

{..., -3, -2, -1, 0}

The only difference between this set and the set of negative integers is the inclusion of 0.

Integers:

{..., -3, -2, -1, 0, 1, 2, 3, ...}

This set includes the negative integers, 0, and the positive integers.

Rational Numbers:

The set of integers and fractions.

Real Numbers:

The set of rational and irrational numbers.

We can define a rational number

in terms of decimal representation.

We classify decimals

as terminating, repeating,

or nonrepeating.

A terminating decimal ends.

A repeating decimal has a block of digits that repeats indefinitely.

A nonrepeating decimal does not have a block of digits that repeats indefinitely

and does not terminate.

If the decimal terminates,

it is a rational number.

If a decimal has a block of digits that repeats indefinitely,

it is a rational number.

A decimal that does not have a block of digits

that repeats indefinitely

is an irrational number.

Also, a decimal that doesn't terminate

is irrational.

An *irrational number*

is a number that's neither whole nor a fraction.

Again, irrational numbers

have a *non-repeating decimal*.

To fill in any blanks,

it doesn't matter how many decimal places you write down;

you can always write more.

Furthermore, the digits in this decimal never become repetitive

or fall into any pattern.

Together,

rational and irrational numbers

make the set of *real numbers.*

This set contains every point on the real number line.

Absolute Value

The *absolute value*

of a number

is the distance between the number

and 0 on the real number line.

Absolute value

is denoted by vertical bars.

Thus, we write

$|2| = 2$

$|-3| = 3$

$|0| = 0$

The Basic Four Operations

When math is brought up,

most people usually think of four words:

addition, subtraction, multiplication, and division.

Throughout this book,

I will refer to these operations

as the *Basic Four* operations.

Addition: Adding Things

Addition is the first operation we will discuss.

Addition is all about adding things together.

This operation uses just one sign - the plus sign (+).

Addition of Real Numbers

Two Positive Numbers:

The sum of two positive real numbers

is the sum of their absolute values.

Two Negative Numbers:

The sum of two negative real numbers

is the opposite of the sum of their absolute values.

One Positive and One Negative Number:

The sum of a positive real number

and a negative real number

can be found by subtracting

the smaller absolute value

from the larger absolute value

and giving the result the sign of the original number

that has the arger absolute value.

If the two numbers have the same absolute value,

then their sum is 0.

Zero and Another Number:

The sum of 0 and any real number

is the real number itself.

Examples:

(a) (-5) + (-3)

(-5) + (-3) =

$-(|-5| + |-3|) = -(5 + 3) = -8$

(b) 15 + (-3)

15 + (-3) =

$(|15| - |-3|) = (15 - 3) = 12$

(c) $-72.4 + 72.4$

$-72.4 + 72.4 = 0$

(d) $12 + 9$

$12 + 9 = |12| + |9| = 12 + 9 = 21$

REMEMBER -

When adding two numbers together,

the two numbers are called *addends*,

and the result is the *sum*.

TIP -

You can form an equivalent subtraction problem when adding a negative number.

7 + (-4) is the same as 7 - 4.

Subtraction: Taking Things Away

Subtraction is the second operation we will discuss,

and it's normally the second operation you learn about.

As is the case with addition,

subtraction isn't that hard.

It uses just one sign: the minus sign (-).

Subtraction of Real Numbers

If a and b are both real numbers,

then

$$a - b = a + (-b)$$

It could be helpful for you to read

$a - b = a + (-b)$

as "a minus b is equivalent to a plus the opposite of b."

In other words,

you can form an equivalent addition problem

out of any subtraction problem.

Examples:

(a) 7 - 12

$$7 - 12 = 7 + (-12) = -5$$

(b) (-5) - (-12)

$$(-5) - (-12) = (-5) + 12 = 7$$

(c) 15 - (-15)

15 - (-15) =

15 + 15 = 30

REMEMBER -

When you subtract one number

from another number,

the result is the *difference*.

TIP -

When subtracting a negative number,

you can form an equivalent addition problem.

In other words, 3 - (-3) is the same as 3 + 3.

Multiplication

We can consider the multiplication

of whole numbers as repeated addition.

For example,

5 X 3 means *add 5 to itself 3 times:*

5 + 5 + 5 = 15

4 X 8 means *add 4 to itself 8 times:* 4 + 4 + 4 + 4

+ 4 + 4 + 4 + 4

Multiplication of Real Numbers

1. The product of two positive

or two negative real numbers is the product

of their absolute values.

2. The product of a positive real number and a negative real number

(either order)

is the opposite of the product of their absolute values.

3. The product of zero and any real number is zero.

(a) (-5)(-5)

$(-5)(-5) = |-5| \times |-5| = 5 \times 5 = 25$

(b) (8)(-8)

$(8)(-8) = -(|8| \times |-8|) = -(8 \times 8) = -64$

(c) 0(3) = 0

REMEMBER:

When multiplying two numbers,

the numbers that you're multiplying are called *factors*.

The result is the *product*.

In the previous example, 4 and 8 are the factors

and 32 is the product.

When you are first introduced to multiplication,

you were probably taught to use the *times* sign (x).

However,

other areas of math

use different symbols to indicated multiplication.

Using the Dot

In nath beyond basic arithmetic,

the · symbol replaces the X symbol.

For example,

5 · 5 = 25 means 5 X 5 = 25

10 · 11 = 110 means 10 X 11 = 110

That's it!

Just use the · symbol

anywhere you'd use the standard times sign (X).

Parentheses

REMEMBER -

In other areas of mathematics,

using parentheses *without* another operator

indicates multiplication.

For example,

3(5) = 15 means 3 X 5 = 15

(10)12 = 120 means 10 X 12 = 120

(8)(8) = 64 means 8 X 8 = 64

Division: Dividing Things

The last operation we will discuss is division.

Division literally means splitting things up.

For examle,

suppose you're a parent

buying a pizza for your two children to split.

You want the children to split the 8 slices of pizza evenly

so that each child gets the same amount

(don't want to cause an argument, right?).

Each child gets four pieces of pizza.

This problem indicated that

8 ÷ 2 = 4

As with multiplication,

division can also use different signs:

the *division sign* (÷)

and the *fraction slash* (/) or *fraction bar* (—).

REMEMBER:

When dividing one number by another,

the first number is called the *dividend*.

The second number is called the *divisor*.

The result is the *quotient*.

If you were thinking of the previous example,

the dividend is 8,

the divisor is 2,

and the quotient is 4.

Division of Real Numbers

1. The quotient of two positive or two negative real numbers

is the quotient of their absolute values.

2. The quotient of a positive real number

and a negative real number

or of a negative real number

and a positive real number

is the opposite of the quotient of their absolute values.

3. The quotient of zero and any nonzero reasl number is zero.

4. The quotient of any nonzero real number and zero is undefined.

Examples

(a) (-16) ÷ (-4) = |-16| ÷ |-4| = 16 ÷ 4 = 4

(b) 16 ÷ (-4) = -(|16| ÷ |-4|) = -(16 ÷ 4) = -4

(c) 12 ÷ 0 = undefined

(d) 0 ÷ 12 = 0

Properties of the Basic Four Operations

When you know how to do the Basic Four operations -

addition, subtraction, multiplication, and division -

you're ready to get to know

different properties of these operations.

Properties

are features of the Basic Four operations

that apply

no matter which numbers you're dealing with.

Inverse Operations

Each of the Basic Four operations

has an *inverse*-

an operation that undoes it.

Addition and subtraction are inverse operations;

addition undoes subtraction,

and subtraction undoes addition.

In the same way,

multiplication and division are inverse operations.

Here are two inverse equation examples:

185 - 10 = 175

175 + 10 = 185

3 X 6 = 18

18 ÷ 6 = 3

Commutative Operations

Addition and multiplication are commutative operations.

Commutative means that you can switch the oder of the numbers around

without changing the result.

This propery of addition and multiplication

is called the *commutative property*.

For example,

2 + 3 = 5 is the same as 3 + 2 = 5

2 X 8 = 16 is the same as 8 X 2 = 16

Commutative Property of Addition

If a and b are real numbers,

then

$a + b = b + a$

Commutative Property of Multiplication

If a and b are real numbers, then

$ab = ba$

TIP -

In contrast,

subtraction and division are NOT

commutative operations.

They are *noncommutative* operations.

When you switch around the order of the numbers,

the result DOES change.

For example,

6 - 3 = 3, but 3 - 6 = -3

Associative Operations

Both addition and subtraction

are *associative operations*.

This means you can group them differently

without changing the result.

This property of addition and multiplication

is also called the *associative property*.

Here's an example of how addition is associative.

Suppose you would like to add 3 + 2 + 6.

You can solve this problem in two ways:

(3 + 2) + 6 =

(5) + 6

= 11

or

3 + (2 + 6) =

3 + (8) =

11

And here's an example of how multiplication is associative.

Suppose you'd like to multiply

5 X 3 X 4.

You can solve this problem in two ways:

(5 X 3) X 4

= (15) X 4

= 60

or

5 X (3 X 4) =

5 x (12) =

60

In contrast,

subtraction and division are nonassociative operations.

This means that grouping them in different ways will change the result.

Associative Property of Addition

If a, b, and c are real numbers, then

$(a + b) + c = a + (b + c)$

Associative Property of Multiplication

If a, b, and c are real numbers, then

(ab)c = a(bc)

Distributive Property

Distributive Property

If a, b, and c are real numbers, then

a(b + c) = ab + ac

The distributive property ties together

the operations of addition and multiplication.

We say that *multiplication distributes over addition*.

For example, 2(3 + 5) = 2 (3) + 2 (5).

Because b - c = b + (-c),

it follows that *multiplication also distributes over subtraction*.

This can be expressed as a(b-c) = ab - ac.

For example, 3 (10 - 2) = 3 (10) - 3 (2).

For example,

suppose you want to multiply 10 X 100.

You can multiply them out,

but distribution gives you a different way to think about the problem.

You might find this way easier.

Because 100 = 50 + 50,

you can split this prob lem into two easier problems as follows:

$= 10 \text{ X } (50 + 50)$

$= (10 \text{ X } 50) + (10 \text{ X } 50)$

$= (500) + (500) = 1{,}000$

Other Operations:

Exponents and Square Roots

In this section,

I'll be introducing you to two new operations

that you will need as you move on with math:

exponents and square roots.

As with the Basic Four operations,

these two operations

take numbers

and tweak them in different ways.

Exponents

Exponents (also reffered to as *powers*)

are shorthand for repeated multiplication.

For example, 2^3 means to multiply 2 by itself 3 times.

To do so, use the following notation:

$$2^3 = 2 \text{ X } 2 \text{ X } 2 = 4 \text{ X } 2 = 8$$

In the preceding example,

2 is the *base number*

and 3 is the *exponent*.

You can read 2^3 as "two to the third power"

or "two to the power of 3"

(or even "two cubed,"

which has to do with the formula for finding the volume of a cube-

see C11 for details).

TIP -

When the base number is 10,

figuring out the exponent is a breeze.

Just write down a 1 and that many 0s after it:

10^2 = 100 (1 with two 0s)

10^7 = 10,000,000 (1 with seven 0s)

10^{10} = 10,000,000,000 (1 with ten 0s)

The most common exponent is the number 2.

When you take any whole number to the power of 2,

the result is a square number.

For that reason,

taking a number to the power of 2

is called *squaring* that number.

You can read 3^2 as "three squared," 4^2 as "four squared," and so on.

REMEMBER -

Any number raised to the 0 power equals 1.

So 1^0, 37^0 and $999{,}999^0$

are equivalent, or equal.

Roots

Earlier in this chapter,

in "Properties of the Basic Four Operations,"

I showed you how addition and subtraction are inverse operations.

I also showed you how multiplication and division are inverse operations.

In a similar way,

roots are the inverse operation of exponents.

The most common root is the square root.

A *square root* undoes an exponent of 2.

For example,

$4^2 = 4 \text{ X } 4 = 16$,

so $\sqrt{16} = 4$

You can read the symbol $\sqrt{}$ as either "the square root of"

or as "radical."

So read $\sqrt{16}$ as either

"the square root of 16" or "radical 16."

Finding Facors

In this section,

I introduce you (or reintroduce you)

to factors and multiples.

When one number is a *factor* of a second number,

the second number is a *multiple* of the first number.

For example, 20 is divisible by 4,

so 4 is a factor of 20

and 20 is a multiple of 4.

Generating Factors

It's easy to tell whether a number

is a factor of a second number.

Just divide the second number by the first one.

If it divides evenly (that is, with no remainder),

the number is a factor;

otherwise, it's not.

As an example,

suppose you'd like to know

whether 8 is a factor of 64.

Because 8 divides 64 without leaving a remainder, 8 is a factor of 64.

This method works

no matter how larger the numbers are.

REMEMBER -

The *greatest factor* of any number

is the number itself.

You can always list all of the factors of a number

because you have that stopping point.

Here's how to list all the factors of a number:

1. **Begin the list with 1.**

Leave space for other numbers,

and end the list with the number itself.

Supposed you'd like to list all the factors of the number 18.

Following these steps,

you'd begin your list with 1 and end it with 18.

2. **Test whether 2 is a factor.**

In other words, see if the number is divisible by 2.

If it is, then add 2 to the list.

For example, 18 ÷ 2 = 9,

so add 2 and 9 to the list of factors of 18.

3. **Test the number 3 in the same way.**

You can see that 18 ÷ 3 = 6,

so add 3 and 6 to the list.

4. **Continue testing numbers until the beginning of the list meets the end of it.**

Check every number between to see

if it's evenly divisible.

If it is,

that number is also a factor.

You get remainder when dividing 18 by 4 or 5,

so the complete list of factors of 18

is 1, 2, 3, 6, 9, and 18.

REMEMBER -

A *prime number* is divisible only by 1 and itself-

for example, the number 7

is only divisible by 1 and 7.

In contrast,

a *composite number*

is divisible by at least one number other than 1 and itself-

e.g.,

te number 9 is divisible not only by 1 and 9

but also by 3.

A number's *prime factors*

are the set of prime numbers (including repeats)

that equal that number when multiplied together.

Greatest Common Factor (GCF)

The *greatest common factor* (GCF)

of a set of numbers

is the largest number that's a factor of all those numbers.

For example,

the GCF of the numbers 4 and 6 is 2,

because 2 is the greatest number that's a factor of both 4 and 6.

REMEMBER -

To find the GCF of a set of numbers,

list all the factors of each number,

as I discussed in "Generating Factors."

The greatest factor appearing on every list is the GCF.

For example, supposed you'd like to find the GCF of 28, 42, and 70.

Start by listing the factors of each:

Factors of 28: 1, 2, 4, 7, 14, 28

Factors of 42: 1, 2, 3, 6, 7, 14, 21, 42

Factors of 70: 1, 2, 5, 7, 10, 14, 35, 70

The largest factors that appears on all three lists is 14;

thus, the GCF of 28, 42, and 70 is 14.

Multiples

Multiples tend to be larger numbers than factors.

Keep reading for info on finding multiples

and identifying the least common multiple of a set of numbers.

Generating Multiples

The earlier section "Find Factors"

tells you how to find all the factors of a number.

Finding all the factors is possible because a number's factors

are always less than or equal to the number itself.

So no matter how large a number may be,

it always has a *finite* (limited) number of factors.

Unlike factors,

multiples of a number

are greater than or equal to the number itself.

(The only exception is 0,

which is a multiple of every number.)

Because of this,

the multiples of a number go on forever-

that is, they're *infinite*.

Nevertheless,

generating a partial list of multiples

for a number is simple.

REMEMBER -

To list the multiples of any number,

write down that number

and then multiply it by 2, 3, 4, and so on.

For example, here are a few positive multiples of 7:

7 14 21 28 35 42

As you can see,

the list of multiples

is simply part of the multiplication table for the number 7.

Finding the Least Common Multiple (LCM)

REMEMBER -

The *least common multiple* (LCM)

of a set of numbers is the lowest positive number

that's a multiple of every number in that set.

To find the LCM of a set of numbers,

take each number in the set and jot down a list of the first several multiples in order.

The LCM is the first number that appears on every list.

TIP! -

When looking for the LCM of two numbers,

start by listing multiples of the higher number.

Stop this list when the number of multiples you've written down

equals the lower number.

Then start listing multiples of the lower number

until one of them matache a number in the first list.

For example,

suppose you'd like to find the LCM of 4 and 6.

Begin by listing multiples of the higher number-6.

In this case,

list out just four of these multiples

because the lower number is 4.

Multiples of 6: 6, 12, 18, 24, ...

Now, start listing multiples of 4:

Multiples of 4: 4, 8, 12, ...

Because 12 is the first number

to appear on both lists of multiples,

12 is the LCM of 4 and 6.

Chapter 2

Evaluating Arithmetic Expressions

In This Chapter

- Learn about equations, expressions, and evaluating
- Operate in the correct order
- Handle expressions that contain exponents
- Evaluate expressions with parentheses

In this chapter,

we'll be discussing

three important concepts:

equations, expressions, and evaluation.

You more than likely already know

what an equation is.

An *equation* (*equality*) is a mathematical statement in which two symbols,

or groups of symbols, are names for the same number.

In other words,

it's a math statement with an equal sign (=)-

for example, 3 + 2 = 5.

An *expression*

is a string of mathematical symbols

palced on a side of an equation-

for example, 3 + 2.

Evaluating is the process of finding

out the *value* of an expression as a number-

for example, finding out that the expression 3 + 2

is equivalent to the number 5.

Equations, Expressions, Evaluations

You should find equations, expressions, and evaluations

to be quite familiar.

You've more than likely been using them for quite a while.

You have to add up the cost of items when shopping,

you might have to balance your checkbook,

and I'm pretty sure that you've tried to figure out the area of something before.

When doing these things,

you're evaluating expressions and setting up equations.

In this section,

it's my goal to help you understand this stuff.

I want to simplify the process

and help you guys understand this stuff without having a hard time!

Equations

As previously stated,

an *equation* (*equality*)

is a statement in which two symbols-or groups of symbols-

are names for the same number.

In other words, it's a statement with an equal sign.

The equation is such an important concept in mathematics

because it allows you take take a bunch of complicated information

and get a single number out of it.

Mathematical equations come in lots of different forms:

arithmetic equations, algebraic equations, differential equations,

partial differential equations, Diophantine equations, and so on.

In this book, we will just be looking at two types:

arithmetic equations and algebraic equations.

In this chapter,

we will just be discussing arithmetic equations.

Arithmetic equations are equations involving numbers,

the Basic Four operations, and other basic operations

we looked at in Chapter 1

(absolute value, exponents, roots).

Here are a few examples of simple arithmetic equations:

3 + 2 = 5

3 X 4 = 12

18 ÷ 3 = 6

And here are a few examples of more-complicated

arithmetic equations:

1,000 - 1 - 1 - 1 = 997

(1 X 1) + (2 X 2) = 5

$$4^2 - \sqrt{256} = (791 - 842) \cdot 0$$

Expressions

An *expression* is a string of mathematical symbols

placed on a side of an equation.

Just like equations, mathematical expressions come in lots of forms.

In this chapter, the focus will be on arithmetic expressions.

Arithmetic expressions are expressions that contain

numbers, the Basic Four operations, and other basic operations (see C1).

Here are a few examples of simple expressions:

3 + 2

-15 + (-1)

18 ÷ 2

And here are a few examples of more-complicated expressions:

(50 - 25) ÷ 5

100 + 10 - 5 X 3

$$\sqrt{441} + \left|-2^3\right|$$

Evaluation

At the core of the word *evaluation* is theword *value*.

When evaluating something,

you are looking for its value.

We also refer to evaluating an expression

as *simplifying*, *solving*, or *finding the value of* an expression.

The words can vary,

but the idea remains the same:

get a single number out of

a string of math symbols and numbers.

As just stated,

we want to simplify an arithmetic expression

to a single value-

in other words,

find the number it's equal to.

As an example,

evaluate the following arithmetic expression:

6 * 5

How? Get the single number:

30

Putting Things Together

Let's talk about how equations, expressions, and evaluations

are related.

Evaluation allows you to take an *expression*

and reduce it to a single numerical value.

Then, you can make an equation.

Just use the equal sign to connect the expression and the number.

As an example,

here's an expression containing four numbers:

2 + 3 + 4 + 5

When you *evaluate* it,

you are looking to get a single number out of it:

14

Now, let's make an *equation*.

Just connect the expression and number

with an equal sign:

2 + 3 + 4 + 5 = 14

The Order of Operations

When you first started using computers,

did you ever try using the keyboard

before realizing it wasn't connected to the computer?

If so, you more than likely discovered this simple rule:

1. **Connect keyboard to computer.**
2. **Use keyboard.**

Thus, you have an order of operations.

The keyboard must be connected before using it.

This is simple, right?

In this section,

we'll cover a similar set of rules called the *order of operations*

(also known as the *order of precedence*).

Don't let the long name frighten you!

The order of operations is just a set of rules

to make sure you get things done in the correct order.

REMEMBER -

Evaulate arithmetic expressions from left to right.

Use the following order of operations:

1. **Parentheses**

2. **Exponents**

3. **Multiplication and Division**

4. **Addition and Subtraction**

It's ok if you can't remember all of this list at the moment.

I'll break it down for you in the remaining sections of ths chapter.

I'll start from the bottom and work towards the top, as follows:

In "Order of Operations and the Basic Four Expressions,"

I show Steps 3 and 4-

how to evaluate expressions

with any combo of addition, subtraction, multiplication, and division.

In "Order of Operations in Expressions with Exponents,"

I show you how Step 2 fits in-

how to evaluate expressions with the Basic Four operations

plus exponents, square roots, and absolute values.

In "Order of Operations in Expressions with Parentheses,"
I show you how Step 1 fits in-

how to evaluate all the expressions I explain *plus*

expressions with parentheses.

Order of Operations and the Basic Four Expressions

As I covered earlier in this chapter,

evaluating an expression

just means getting a number out of it.

Now,

I get you started on evaluating expressions

that contain any combination of the Basic Four operations-

addition, subtraction, multiplication, and division.

(For more on the Basic Four Operations, see Chapter 1.)
Generally speaks, the Basic Four expressions

come in three types

as follows:

Types of the Basic Four Expressions

1. Expression: Contains only addition and subtraction

Example: 12 + 5 - 6 - 1 + 3

Rule: Evaluate from left to right.

2. Expression: Contains only multiplication and division

Example: 16 ÷ 3 X 7 ÷ 14

Rule: Evaluate from left to right.

3. Expression:
Contains a combo of addition/subtraction

and multiplication/division (mixed-operator expressions)

Example:

5 + 9 ÷ 3

Rule:

1. Evaluate multiplication and division from left to right.

2. Evaluate addition and subtraction from left to right.

In this section, I'll be showing you how to identify and evaluate each type of expression.

Expressions with Only Addition and Subtraction

Some expressions contain just addition and subtraction.

When this is the case,

the rule for evaluating the expression is quite simple.

REMEMBER:

When an expression contains only addition and subtraction,

evaluate it step by step from left to right.

For example, suppose you want to evaluate this expression:

18 - 5 + 3 - 2

Because the only operations are addition and subtraction,

you can evaluate it from left to right.

Just start with 18 - 5:

= 13 + 3 - 2

As is easy to tell,

the number 13 replaces 18 - 5.

Now the expressions has three numbers

rather than four.

Now, evaluate 13 + 3:

= 16 - 2

This simplifies the expression down to two numbers,

which you can rather easily evaluate:

= 14

So 18 - 5 + 3 - 2 = 14

Expressions with Only Multiplication and Division

Some expressions just contain multiplication and division.

If this is the case,

the rule for evaluating the expression is still pretty simple.

REMEMBER:

When an expression only contains multiplication and division,

evaluate it step by step

from left to right.

Suppose you'd like to evaluate this expression:

9 X 2 ÷ 6 ÷ 3 X 2

Move from left to right,

starting with 9 X 2:

18 ÷ 6 ÷ 3 X 2

= 3 ÷ 3 X 2

= 1 X 2

= 2

Notice that the expression will keep shrinking

a number at a time until all that's left is the result.

So 18 ÷ 6 ÷ 3 X 2 = 2

Here's another quick example:

-2 ÷ 6 ÷ -4

Even though this expression contains some negative numbers,

the only onperations it contains are multiplication and division.

You can evaluate it in two steps.

Move from left to right:

= -12 ÷ (-4)

= 3

Thus -2 ÷ 6 ÷ -4 = 3

Mixed-Operator Expressions

Often, an expression contains

- At least one addition or subtraction operator
- At least one multiplication or division operator

I call these *mixed-operator expressions*.

To evaluate them,

you need some stronger tool.

Here's the rule you need to follow.

REMEMBER:

Evaulate mixed-operator expressions like so:

1. **Evaluate the multiplication and division from left to right.**
2. **Evaluate the addition and subtraction from left to right.**

For example, suppose you'd like to evaluate the following expression:

5 + 3 X 2 + 8 ÷ 4

As is obvious,

this expression contains addition, multiplication, and division.

It is a mixed-operator expression.

To evaluate it,

start out by underlining the multiplication and division in the expression:

$5 + \underline{3 \text{ X } 2} + \underline{8 \div 4}$

Now, evaluate the underlined portion from left to right:

$= 5 + 6 + \underline{8 \div 4}$

$= 5 + 6 + 2$

At this point,

you're left with an expression

that contains just addition.

Evaluate it from left to right:

$= 11 + 2$

$= 13$

Thus $5 + 3 \times 2 + 8 \div 4 = 13$.

Order of Operations in Expressions with Exponents

Here's what you need in order to evaluate expressions

that contain exponents

(see Chapter 1 for more info on exponents).

REMEMBER -

Evaluate exponents from left to right *before*

you start evaluating Basic Four operations

(addition, subtraction, multiplication, and division).

The thing to know here is to turn the expression into a Basic Four expression

and then use what I showed you earlier in "Order of Operations and the Basic Four expressions."

As an example, suppose you'd like to evaluate the following:

$$3 + 3^2 - 6$$

First, evaluate the exponent:

$$3 + 9 - 6$$

At this point,

the expression only contains addition and subtraction.

Evaluate it from left to right in two steps:

$$= 12 - 6$$

$$= 6$$

So $3 + 3^2 - 6 = 6$.

Order of Operations in Expressions with Parentheses

In math, parentheses - ( ) - are often used to group together parts of an expression.

When you're evaluating expressions,

here's what you need to keep in mind

about parentheses.

REMEMBER:

To evaluate expressions that contain parentheses,

do the following:

1. **Evaluate the contents of the parentheses, from the inside out.**

2. **Evaluate the rest of the expression.**

Basic Four Expressions with Parentheses

Suppose you'd like to evaluate (1 + 15 ÷ 5) + (3 - 6) X 5.

This expression has to sets of parentheses,

so evaluate these from left to right.

Notice that the first set of parentheses contains a mixed-operator expression.

Evaluate this in two steps,

starting with the division:

(1 + 3) + (3 - 6) X 5

= 4 + (3 - 6) X 5

Now, evaluate the contents of the second set of parentheses:

= 4 + (-3) X 5

Now you have a mixed operator expression,

so evaluate the multiplication (-3 X 5) first.

This gives you this:

= 4 + (-15)

Lastly, evaluate the addition:

= -11

So, $(1+15\div5) + (3-6) \times 5 = -11$.

Expressions with Exponents and Parentheses

Try out the following example,

which contains both exponents and parentheses:

$$1 + (3 - 6^2 \div 9) \times 2^2$$

Start by working *only* on what's inside of the parentheses.

The first thing to evaluate here is the exponent, 6^2:

$$= 1 + (3 - 36 \div 9) \times 2^2$$

Continue working inside the parentheses by evaluating the division $36 \div 9$:

$$= 1 + (3-4) \times 2^2$$

Now, let's get rid of the parentheses completely:

$$= 1 + (-1) \times 2^2$$

At this point,

what's left is an expression with an exponent.

This expression takes three steps,

starting with the exponent:

$$= 1 + (-1) \times 4$$

$= 1 + (-4)$

$= -3$

So, $1 + (3 - 6^2 \div 9) \times 2^{2} =$ -3.

Exponents with Parentheses Raised to an Exponent

Sometimes, the entire contents of the parentheses

are raised to an exponent.

If this is the case,

evaluate the contents of the parentheses

before evaluating the exponent.

Here's an example:

$(5 - 3)^3$

First, evaluate 5 - 3:

$= 2^3$

Now, let's evaluate the exponent:

$= 2 \times 2 \times 2 = 4 \times 2 = 8$

Every once in a while,

the exponent itself contains parentheses.

As usual,

evaluate what's in the parentheses first.

For example,

$21^{(19 + \underline{3 X -6})}$

This time,

the smaller expression inside the parentheses

is a mixed-operator expression.

I underlined the parts

that needs to be evaluated first:

$21^{(19 + -18)}$

Now you can finish off what's inside parentheses:

$= 21^{1}$

At this point,

all that's left is a pretty simple exponent:

= 21

So, $21^{(19 + 3 X -6)} = 21$.

Note:

Technically speaking,

there's no need to put parentheses around the exponent.

If you see an expression in the exponent,

just treat it as if it had parentheses around it.

In other words,

$21^{19 + 3 X -6}$ is the same thing as $21^{(19 + 3 X -6)}$.

Expressions with Nested Parentheses

Occasionally,

an expression has *nested parentheses*:

on or more sets of parentheses inside another set of parentheses.

At this point,

I give you the following rule for handling nested parentheses:

REMEMBER:

When evaluating an expression with nested parentheses,

evaluate what's inside the *innermost* set of parentheses first

and work your way toward the *outermost* parentheses.

For example, suppose you want to evaluate the following expression:

$2 + (9-(\underline{7-3}))$

I underlined the contents of the innermost set of parentheses,

so evaluate these contents first:

$= 2 + (9 - 4)$

Next, evaluate what's inside the remaining set of parentheses:

= 2 + 5

Now, things can be simplified quite easily:

= 7

So, 2 + (9-(7-3)) = 7.

As a final example,

here's an expression that requires everything from this chaper:

$4 + (-7 X (2^{\underline{(5-1)}} - 4 X 6))$

This expression is about as complicated

as you're every likely to see in pre-algebra:

one set of parentheses containing another set,

which contains a third set.

To start off,

I underlined what's deep inside the third set of parentheses.
This is where you begin evaluating:

$= 4 + (-7 X (\underline{2^4 - 4 X 6}))$

Now, what's left is one set of parentheses inside another set.

Again, work from the inside out.

The smaller expression here is $2^4 - 4 X 6$,

so evaluate the exponent first,

then the multiplication, and finally the subtraction:

$$= 4 + \left(-7 \cdot \left(\underline{16 - 4 \cdot 6}\right)\right)$$
$$= 4 + \left(-7 \cdot \left(\underline{16 - 24}\right)\right)$$
$$= 4 + \left(-7 \cdot -8\right)$$

Only one more set of parentheses left:

= 4 + 56

At this point, the answer is obvious:

= 60

Therefore, 4 + (-7 X ($2^{(5-1)}$ - 4 X 6)) = 60.

As I said earlier in this section,

you won't see a problem much more challenging

than this at this stage of mth.

Write it down on a piece of paper

and try solving it with this book closed.

<u>Chapter 3</u>

Handling Word Problems

In This Chapter

- Understanding the four steps for solving word problems
- Forming simple word equations to grasp important information
- Plugging numbers to solve the problem when working with word equations
- Approaching more-complex word problems with confidence

The very mention of word problems-

or *story problems*,

as they're sometimes called,

is enough to bring back frightening memories

of anxiety into the core of the average math student.

Lots of people would rather walk across a bed full of angry snakes

than "figure out how many bushels of corn Farmer Smith picked"

or "help Aunt Blackburn decide how many cookies to bake."

I can't stress enough how important

of a skill solving word problems is.

They help you understand how to set up equations

in real-life situations.

This makes the math more useful.

In this chapter,

I show you how to solve word problems

using four basic steps.

After you understand how to approach the easier problems,

I show you how to apparch the more-complex problems.

Some of these problems might have longer numbers to calculate,

and some could have more-complicated stories.

In either case, I'll be there with you!

We will walk through the process in simple steps.

Basic Word Problems

Generally speaking,

solving a word problem involves four steps:

1. **Read the problem carefully**

and set up *word equations*-

in other words, equations that contain words and numbers.

2. **Substitute numbers in place of words**

whenever possible

to set up a regular math equation.

3. **Use your math skills to solve the equation.**

4. **Be sure to answer the question that the problem asks.**

The majority of this chapter focuses on Step 3.

However, this chapter and Chapters 7 and 11

focus on Steps 1 and 2.

I show you how to break down a word problem

sentence by sentence,

write down only the information you need to solve the problem,

and then plug in numbers for words to set up an equation.

After you learn how to get a word problem out of an equation,

the hard part is out of the way.

Then, you can use the rest of what this book teaches you

to do Step 3-

solve the equation.

From that point, Step 4 is quite easy.

At the end of each example, I'll make sure you know what you're doing.

Getting Word Equations Out of Word Problems

The first step

to solving a word problem is actually reading the problem.

You need to juice the important information out of word problems

and put it into a useful form.

In this section, I show you how to do just that!

Writing Down Information as Word Equations

Most word problems give you information about numbers,

telling you how much, how many, how fast, how long, how big,

and so on.

Here are a few examples:

Devin is baking 18 cookies.

The width of the house it 120 feet.

If the local train is going 25 mph...

In order to actually solve the problem,

you need this information.

Paper is cheap;

don't be scared to put it to use!

Have a piece of scrap paper close by

and jot down notes as you read through a word problem.

For example,

here's how you can jot down "Devin is baking 18 cookies":

Devin = 18

Here's how to note that "the width of the house is 120 feet":

Width = 120

The third example informs you,

"If the local train is going 25 miles per hour..."
So you can jot down the following:

Local = 25

REMEMBER:

Don't let the word *if* confuse you.

Whena problem say "If so-and-so is true..."

and asks you a questions,

assume it *is* true.

Use this info to answer the question.

When jotting down information in this type of way,

you're really just forming *word equations*.

A word equation has an equal sign like a mathematical equation,

but it contains both words and numbers.

Forming Word Equations from More-Complex Statements

When you start working with word problems,

you more than likely noticed something:

certain words and phrases show up repeatedly.

For example,

Devin has four fewer cookies than Nicky.

The height of a house is half as long as its width.

The express train is moving three times faster than the local train.

You've probably seen statements such as these before.

When you first started doing math, and working with word problems,

you more than likely were introduced to these.

They look like English,

but they're actually math.

You can represent each of these types of statements

as word equations that use the Basic Four operations.

Let's take a second look at the first example:

Devin has four fewer cookies than Nicky.

You don't know the number of cookies than either Devin or Nicky have.

But, you do know that these numbers are related.

You can express the relationship like so:

Devin = Nicky - 4

This word equation is definitely shorter than the statement it originated from.

And, as you will see in the following section,

word equations can easily be converted into the math

that you need to solve the problem.

Here's another example:

The height of a house is half as long as its width.

You do not know the width or height of the house,

but you do know that these numbers are related

(as before).

You can express this relationship between the width and height of the house

as the following word equation:

Height = Width ÷ 2

With the same critical thinking skills,

you can express "The express train is moving three times faster than the local train"

as the following word equation:

Express = 3 X Local

As you can tell,

each example allows you to form as word equation

using one of the Basic Four operations-

addition, subtraction, multiplication, and division.

What's the Problem Asking

At the very end of a word problem,

you will often find the question needed to answer the problem.

You can use word equations to clarify the question

so you know right from the beginning what you're seeking.

For example, you can write the question,

"Altogether, how many cookies do Devin and Nicky have?" as

Devin + Nicky = ?

You can write the question "How tall is the house? as

Height = ?

Finally,

you can reword the question

"What's the difference in speed

between the express train and the local train?"

in this way:

Express - Local = ?

Substituting Numbers for Words

After you have a bunch of word equations wrote out,

you have the tools you need in a form you can work with.

Now,

solve the problem by plugging numbers from one word equation into another.

In this section,

I show you just how to use those word equations you've been working on.

Example: Amount of Cookies

Some problems involve only simple addition or subtraction.

Here's a good example:

Devin has four fewer cookies than Nicky.

Nicky has 9 cookies.

Altogether,

how many cookies do Devin and Nicky have?

Here's what you have already:

Nicky = 9

Devin = Nicky - 4

Plugging in the information

gives you this:

Devin = 9 - 4 = 5

The problem is asking you to figure outhow many cookies these two people have together.

In other words,

you need to figure out the following:

Devin + Nicky = ?

Just plug in the numbers,

subsituting 5 for Devin and 9 for Nicky:

~~Devin~~ 5 + ~~Nicky~~ 9 = 14

So, Devin and Nicky have 14 cookies.

Example: Our house at the End of the Road

There are times that a problem

can contain relationships in which

you are required to use multiplication and division.

Here's an example:

The height of a house is half as long as its width.

The width of the house is 120 feet.

How tall is the house?

You already have a head start from what you learn earlier

in "Forming Word Equations from More-Complex Statements":

Width = 120

Height = Width ÷ 2

You can substitute information as follows,

plugging in 120 for the *width*:

Height = ~~Width~~ 120 ÷ 2 = 60

So, you know that the height of the house has to be 60 feet.

Approaching More-Challenging Word Problems

The skills I discussed previously in "Basic Word Problems"

are important for solving any word problem.

They give you a guideline to go by,

and they make the process of solving word problems much simpler.

And, what's more,

you can use those same skills to work your way through more-complex problems.

Word problems become more-complex when

- The calculations become harder. (For example, instead of a shirt costing $30, now it costs $29.95.)
- There's an increase in the amount of information in the problem. **(**For example, now you have four people with cookies instead of two.)

Don't let problems like these intimidate you.

In this section,

I show you how to put your new problem-solving skills to use

and solve these more-difficult word problems with ease!

When Numbers Increase

A lot of problems that look intimidating

aren't really that much more difficult

than problems we looked at in previous sections.

For example, consider this problem.

Aunt Ellen has $735.85 hidden in her safe,

and Uncle Ray has $235.21 less than Aunt Ellen has.

How much money do these two people have altogether?

Even though the numbers have increased,

the principle is still the same.

Start by reading the first line:

"Aunt Ellen has $735.85 hidden in her safe."

This text is just info to jot down as a simple word equation:

Ellen = $735.85

Continue reading, and you find:

"Uncle Ray has $235.21 less than Aunt Ellen has."

It's another statement that can be convered into an equation:

Ray = Ellen - $235.21

Now, plug in the number $735.85 where you see Aunt Ellen's name in the equation:

Ray = ~~Ellen~~ $735.85 - $232.21

So far, these big numbers haven't caused much trouble!

At this point, though,

let's stop and do the subtraction:

$735.85 - $232.21 = $503.64

Now, you can jot down this information as always:

Ray = $503.64

The question at the end of the problem

wants you to figure out how much money the two people have altogether.

Here's how to represent this problem as a word equation:

Ellen + Ray = ?

Then, Plug information into this equation:

~~Ellen~~ $735.85 + ~~Ray~~ $503.64 = ?

Again,

because the numbers have went up,

let's stop and do the math:

$735.85 + $503.64 = $1,239.49

So, altogether,

Aunt Ellen and Uncle Ray have $1,239.49.

As you have more than likely noticed,

the basic concept for solving this type of problem

is the same for solving simpler problems

such as those in earlier sections.

The only real difference here is that you have to stop and do some addition and subtraction.

More Information

When the going gets tough,

word equations become even more helpful!

Here's a word problem that specifically designed

to try to intimidate you-

but, with your new skills,

I think you're ready for it:

Four men collected money to save the endangered Reelfoot Lake.

Johnny collected $160,

Timmy collected $50 more than Johnny,

Calvin collected twice as much as Timmy,

and together, Calvin and Josh collected $700.

How much money did the four money collect altogether?

If you try to solve this problem in your head,

you'll more than likely get confused.

Instead, just take it line by line.

Also, use word equations as you've worked with throughout this chapter.

First, "Johnny collected $160."

So, jot down the following:

Johnny = 160

Next, "Timmy collected $50 more than Johnny," so write

Timmy = Johnny + 50

After that, "Calvin collected twice as much as Timmy":

Calvin = Timmy X 2

And, finally, "together, Calvin and Josh collected $700":

Calvin + Josh = 700

That's every bit of information that the problem gives you.

So, now, you can start working with it.

Johnny collected $160, so you can subsitute 160 anywhere you find Johnny's name:

Timmy = ~~Johnny~~ 160 + 50 = 210

Now, you know how much Timmy collected.

Plug this info into the next equation:

Calvin = ~~Timmy~~ 210 X 2 = 420

This equation tells you how much Calvin collected, so you can substitute this number into the last equation:

~~Calvin~~ 420 + Josh = 700

To solve this problem,

change it from addition to subtraction.

Use inverse operations-

as shown in Chapter 1:

Josh = 700 - 240 = 280

Now that you knmow how much money each man collected,

you can easily answer the question at the end of the problem:

Johnny + Timmy + Calvin + Josh-= ?

You can plug this information in quite easily:

~~Johnny~~ 160 + ~~Timmy~~ 210 + ~~Calvin~~ 420 + ~~Josh~~ 280 = 1,070

So, you can tell that the four men collected $1,070 altogether.

Putting Things Together

Here's a final example putting everything together

from this chapter.

Try writing down this problem and working through it step by step.

If you get stuck, come back to this chapter.

When you can solve it from beginning to end.

with the book closed,

you'll have a good grasp of how to solve word problems:

On a recent shopping trip,

Travis bought six shirts for $19.95 each

and two pairs of jeans for $34.60 each.

Hen then bought a jacket that cost $37.08 less than he paid for both pairs of jeans.

If he paid the cashier with three $100 bills,

how much change did he receive?

You can jot down the following word equations:

Shirts = $19.95 X 6

Jeans = $34.60 X 2

Jacket = Jeans - $37.08

The numbers in this problem

more than likely make this problem

too complex to solve in your head.

They required attention:

$$\begin{array}{rr} \$19.95 & \$34.60 \\ \underline{\times \quad\; 6} & \underline{\times \quad 2} \\ \$119.70 & \$69.20 \end{array}$$

With this out of the way,

you can fill in more blanks:

Shirts = $119.70

Jeans = $69.20

Jacket = Jeans - $37.08

Now, you can plug in $69.20 for *jeans* to find the cost of the jacket:

Jacket = ~~Jeans~~ $69.20 - $37.08

Again, solve this equation

on a piece of paper:

$$\begin{array}{r} \$69.20 \\ \underline{-\$37.08} \\ \$32.12 \end{array}$$

This equation gives you the price of the jacket:

Jacket = $32.12

Now that you have the price of the shirts, jeans, and jacket,

you can figure out how much Travis spent:

Amount Travis Spent = ~~Shirts~~ \$119.70 + ~~Jeans~~ \$69.20 + ~~Jacket~~ \$32.12

Again, you have another equation to solve:

$$\begin{array}{r} \$119.70 \\ \$69.20 \\ \underline{+\$32.12} \\ \$221.02 \end{array}$$

So, you can jot down the following:

Amount Travis Spent = \$221.02

The problem wants you to figure out how much change Travis got back

from \$300,

so jot this down:

Change = \$300 - Amount Travis Spent

You can now plug in the amount that Travis spent:

Change = \$300 - \$221.02

Do just one more equation:

$$\begin{array}{r} \$300.00 \\ \underline{-\$221.02} \\ \$78.98 \end{array}$$

So, you have the answer:

Change = \$78.98

Therefore,

Travis received \$78.98 in change.

<u>Chapter 4</u>

Working with Fractions

In This Chapter

- Multiplying and Dividing Fractions
- Addition and Subtraction of Fractions in Different Ways
- Mixed Numbers and the Basic Four Operations

There are some important things to take not of

about fractions right from the start.

For example, when the *numerator* (top number)

and the *denominator* (bottom number)

are equal,

then the fraction is equivalent to 1.

When the numerator is less than the denominator,

the fraction is <u>less than</u> 1.

Fractions like these are known as *proper fractions*.

Positive proper fractions

are always between 0 and 1.

However,

a fraction is <u>greater than</u> 1

when the numerator is greater than the denominator.

Fractions that are greater than 1

are known as *improper fractions*.

It's customary to convert an improper fraction

to a mixed number-

especially when it's the final answer to a problem.

A *mixed number* is a combination of a whole number

and a proper fraction added together.

In this chapter,

we'll be focusing on applything the Basic Four operations

to fractions.

I start out with the multiplication and division of fractions.

Multiplying and dividing fractions isn't too difficult.

However, addition and subtraction of fractions is a little bit trickier.

Later in this chapter,

I move forward to mixed numbers.

Again, multiplication and division

shouldn't pose too much of a challenge

because the process is almost the same as multiplication

and division of fractions.

At the very end of this chapter,

we will discuss addition and subtraction of mixed numbers.

By that point,

you should be much more comfortable with fractions

and ready to confront the challenge!

Reducing Fractions

In order to reduce a fraction to its lowest terms,

you must identify a common factor in the numerator and denominator.

A fraction is completely reduced, or in its lowest terms,

when the numerator and denominator no longer share

any common factors.

Here's a fraction that isn't in its lowest terms:

$$\frac{28}{40}$$

Jot down the factors of both the numerator and denominator.

Factors of 28: 1, 2, 4, 7, 28

Facotrs of 40: 1, 2, 4, 5, 8, 10, 20, 40

Find the greatest common factor (GCF).

The largest factor of both numers is 4.

Divide the numerator and denominator by the gcf.

$$\frac{28 \div 4}{40 \div 4} = \frac{7}{10}$$

The numerator and denominator do not share any other common factors,
so the fraction is now fully reduced.

Multiplication and Division of Fractions

One thing that shocks a lot of newcomers
is that multiplying and dividing fractions
is simpler than adding or subtracting them.
For that reason,
I show you how to multiply and divide fractions
before showing you how to add or subtract them.

Multiplying Numerators and Denominators Straight Across

It would be quite nice if everything in life were as simple as
multiplying fractions.
All you need to complete the task is a pen or pencil,
a piece of paper,
and basic knowledge of the multiplication table.

REMEMBER:

Here's how to multiply two fractions:

1. **Multiply the *numerators* (top numbers) together
to get the numerator of the answer.**

2. **Multiply the *denominators* (bottom numbers) together**

to get the denominator of the answer.

For example, here's how to multiply $\frac{2}{5} \cdot \frac{3}{7}$:

$$\frac{2}{5} \cdot \frac{3}{7} = \frac{2 \cdot 3}{5 \cdot 7} = \frac{6}{35}$$

When multiplying fractions,

sometimes you have the opportunity to reduce to the lowest terms

(see the preceding section for all of the details).

Flipping to Divide Fractions

As previously stated,

dividing fractions is just as easy

as multiplying them.

In fact,

you can actually turn the problem into a multiplication problem

when dividing fractions.

REMEMBER:

To divide one fraction by a second fraction,

multiply the frist fraction by the reciprocal of the second.

The *reciprocal* of a fraction

is simple that fraction turned upside down.

For example,

here's how you turn fraction division

into multiplication:

$$\frac{1}{3} \div \frac{4}{5} = \frac{1}{3} \cdot \frac{5}{4}$$

As you can tell,

I turned $\frac{4}{5}$ into its reciprocal - $\frac{5}{4}$ -

and changed the division sign to a multiplication sign.

After that,

just multiply the fractions straight across

as discussed in "Multiplying Numerators and Denominators Straight Across":

$$\frac{1}{3} \cdot \frac{5}{4} = \frac{1 \cdot 5}{3 \cdot 4} = \frac{5}{12}$$

Addition of Fractions

When adding fractions,

one important thing to look for is whether

the denominators (bottom numbers)

are the same.

If they are, then

the addition will be a walk in the aprk.

But, adding fractions with different denominators

becomes a bit more complex.

Sum of Fractions with Same Denominators

REMEMBER:

To add two fractions

that have the same denominator

(bottom number),

add the numerators (top numbers) together

and leave the denominator unchanged.

For example, consider the following problem:

$$\frac{1}{5}+\frac{2}{5}=\frac{1+2}{5}=\frac{3}{5}$$

To add these two fractions,

you need to add the numerators (1 + 2) and keep the denominator (5) the same.

Even if you have to add more than two fractions,

you just add the numerators and leave the denominator unchanged

as long as the denominators are all the same:

$$\frac{1}{17}+\frac{3}{17}+\frac{4}{17}+\frac{6}{17}=\frac{1+3+4+6}{17}=\frac{14}{17}$$

There are times that you'll need to reduce to lowest form.

Take this problem as an example:

$$\frac{1}{4}+\frac{1}{4}=\frac{1+1}{4}=\frac{2}{4}$$

The numerator and denominator are both even,

so you know they can be reduced:

$$\frac{2}{4}=\frac{1}{2}$$

In other cases, the sum of two proper fractions

is an improper fraction.

You get a numerator that's larger than the denominator

when the two fractions add up to more than 1-

as in this case:

$$\frac{3}{7}+\frac{5}{7}=\frac{8}{7}$$

If there's still more that you need to do with the fraction,

leave it as an improper fraction

so that it will be easier to work with.

But, you might need to convert it into a mixed numer

if this is the final answer.

I show you how to do so in "Converting Between Improper Fractions

and Mixed Numbers":

$$\frac{8}{7}=8\div 7=1\text{ r }1=1\frac{1}{7}$$

TRAP:

When two fractions have the same numerator (top number),

don't add them by adding the denominators

and leaving the numerator unchanged.

Adding Fractions with Different Denominators

When the fractions that you want to add

have different denominators, adding them is a little more complex.

REMEMBER:

Here's a method for adding fractions with different denominators:

1. **Cross-multiply the two fractions**

and add the results together to get the numerator of the answer.

Suppose you want to add the fractions

$\frac{1}{3}$ and $\frac{2}{5}$.

To get the numerator of the answer, *cross-multiply*.

To break this down for you,

multiply the numerator of each fraction

by the denominator of the other fraction:

$$\frac{1}{3} + \frac{2}{5}$$
$$1 \cdot 5 = 5$$
$$2 \cdot 3 = 6$$

Add the results to get the numerator of the answer:

5 + 6 = 11

2. **Multiply the two denominators (bottom numbers)**

to get the denominator of the answer.

To get the denominator,

just multiply the denominators of the two fractions:

3 X 5 = 15

The denominator of the answer is 15.

3. **Write your answer as a fraction.**

$$\frac{1}{3}+\frac{2}{5}=\frac{11}{15}$$

TIP:

In some cases,

you might want to add more than one fraction.

The method for doing so is similar,

with one small tweak.
For example,

suppose you want to add $\frac{1}{2}+\frac{3}{5}+\frac{4}{7}$,

1. **Start out by multiplying the *numerator***

of the first fraction by the *denominators* of all other fractions.

$$\frac{\mathbf{1}}{2}+\frac{3}{\mathbf{5}}+\frac{4}{\mathbf{7}}$$
$$(1 \cdot 5 \cdot 7)=35$$

2. **Do the same thing with the second fraction,**

and add this value to the first.

$$\frac{1}{\mathbf{2}}+\frac{\mathbf{3}}{5}+\frac{4}{\mathbf{7}}$$
$$35+(3 \cdot 2 \cdot 7)=35+42$$

3. **Do the same thing with the remaining fraction(s).**

$$\frac{1}{2}+\frac{3}{5}+\frac{4}{7}$$
$$35+42+(4\cdot 2\cdot 5)=35+42+40=117$$

When you're done,

you have the numerator of your answer.

4. **To get the denominator,**

just multiply all of the denominators together:

$$\frac{1}{2}+\frac{3}{5}+\frac{4}{7}$$
$$=\frac{35+42+40}{2\cdot 5\cdot 7}=\frac{117}{70}$$

As previously stated,

you could need to reduce or convert an improper fraction to a mixed number.

In this example,

you just need to convert to a mixed number

(as I explain in "Converting Between Fractions and Mixed Numbers"):

$$\frac{117}{70}=117\div 70=1\text{ r }47=1\frac{47}{70}$$

Subtracting Fractions

Subtracting fractions isn't really too much different

from adding them.

And as with addition,

subtraction is easy when the denominators are the same.

I'll show you a different approach,

the *traditional method*,

for subtracting fractions

when the denominators are different.

Subtracting Fractions with the Same Denominator

When two fractions have the same denominator (bottom number),

here's how you go about subtracting one fraction from the other:

Subtract the numerator (top number)

of the second fraction

from the numerator of the first fraction

and keep the denominator the same.

For example,

$$\frac{3}{5} - \frac{2}{5} = \frac{3-2}{5} = \frac{1}{5}$$

Sometimes,

you might need to reduce:

$$\frac{3}{10} - \frac{1}{10} = \frac{3-1}{10} = \frac{2}{10}$$

The numerator and denominator are both even.

You can use that knowledge to reduce this fraction by a factor of 2

(as I show you earlier in "Reducing Fractions to Lowest Terms"):

$$\frac{2}{10} = \frac{2 \div 2}{10 \div 2} = \frac{1}{5}$$

Unlike addition,

you never get an improper fraction

when subtracting one proper fraction from another.

Subtracting Fractions with Different Denominators

Just like with addition,

you have a choice of methods when subtracting fractions.

In this section,

I'll introduce you to the traditional way

to convert fractions with two different denominators.

You can use this method to add fractions as well.

REMEMBER:

To use the traditional method to subtract fractions

with two different denominators,

follow these steps:

1. **Find the least common multiple (LCM)**

of the two denominators (for more on finding the LCM

of two numbers, see C1).

For example, suppose you want to subtract $\frac{7}{8} - \frac{11}{14}$.

Here's how you go about finding the LCM of 8 and 14.

List the first eight multiples of 14,

and then list multiples of 8 until you find a number that appears

in both lists:

Multiples of 14: 14, 28, 42, 56, 70, 84, 98, 112, ...

Multiples of 8: 8, 16, 24, 32, 40, 48, 56, ...

The LCM is 56.

2. **Increase each fraction to higher terms**

so that the denominator of each equals the LCM (least common multiple).

The denominators of both should be 56:

$$\frac{7}{8} = \frac{7 \cdot 7}{8 \cdot 7} = \frac{49}{56}$$
$$\frac{11}{14} = \frac{11 \cdot 4}{14 \cdot 4} = \frac{44}{56}$$

3. **Substitute the two new fractions for the original ones**

and subtract as I show you earlier in "Subtracting Fractions with the Same Denominator."

$$\frac{49}{56} - \frac{44}{56} = \frac{5}{56}$$

This time, you don't need to reduce.

5 is a prime number,

and 56 isn't divisible by 5.

In some cases, however,

you might need to reduce the answer to lowest terms.

Mixed Numbers

Both of the methods I described earlier in this chapter

work for proper and improper fractions.

Unfortunately,

mixed nunmbers are a different ballpark.

You must figure out how to deal with them

on their own terms.

Converting Between Improper Fractions and Mixed Numbers

Improper fractions are often easier to work with,

but you normally need to have a mixed number

for a final answer.

This section shows you how to convert between mixed numbers

and improper fractions.

Converting to an Improper Fraction

Here's how to convert a mixed number to an improper fraction:

1. **Multiply the denominator (bottom number) of the fractional part**

by the whole number,

and add the result to the numerator.

As an example, suppose you'd like to convert the mixed number $5\frac{2}{3}$ to an

improper fraction.

Multiply 3 by 5

and add 2:

(3 X 5) + 2 = 17

2. **Use the result as your numerator (top nmumber),**

and place it over the denominator you already have.

Place 17 over the denominator:

$$\frac{17}{3}$$

So, the mixed number $5\frac{2}{3}$

is equivalent to the improper fraction $\frac{17}{3}$.

Converting to a Mixed Number

To convert an improper fraction to a mixed number,

divide the numerator by the denominator.

Then, write the mixed number this way:

- The quotient (answer) is the whole-number part.
- The remainder is the numerator.
- The denominator of the improper fraction is the denominator.

For example, suppose you want to convert the improper fraction $\frac{19}{5}$

to a mixed number.

First, divide 19 by 5.

The answer is 3

with a remainder of 4:

19 ÷ 5 = 3 r 4

Then, write the mixed number like follows:

$$3\frac{4}{5}$$

Multiplying and Dividing Mixed Numbers

I can't just give you a direct method

for multiplying and dividing mixed numbers.

The only direction is can give you

is to convert the mixed numbers to improper fractions

and multiply or divide as usual.

REMEMBER:

Here's how to multiply or divide mixed numbers:

1. **Convert the mixed numbers to improper fractions.**

As an example, suppose you'd like to multiply $1\frac{3}{5} \cdot 2\frac{1}{3}$.

First, convert both mixed numbers to improper fractions:

$$1\frac{3}{5} = \frac{5 \cdot 1 + 3}{5} = \frac{8}{5}$$
$$2\frac{1}{3} = \frac{3 \cdot 2 + 1}{3} = \frac{7}{3}$$

2. **Multiply these improper fractions.**

$$\frac{8}{5} \cdot \frac{7}{3} = \frac{8 \cdot 7}{5 \cdot 3} = \frac{56}{15}$$

3. **If the answer is an improper fraction,**

convert it back to a mixed number.

$$\frac{56}{15} = 56 \div 15 = 3 \text{ r } 11 = 3\frac{11}{15}$$

In this case,

the answer is already reduced.

You don't have to reduce it.

Adding and Subtracting Mixed Numbers

One way to add and subtract mixed numbers

is to convert them to improper fractions

and add or subtract them.

However,

you can also work with the fractional and whole-number parts separately-

as I show you in this section.

Adding Two Mixed Numbers

Adding mixed numbers looks a lot like adding whole numbers.

For that reason, some students feel less intimidated

adding mixed numbers than adding fractions.

REMEMBER:

Here's how to add two mixed numbers:

1. **Add the fractional parts using any method you're comfortable with.**

If necessary, reduce.

2. **If the answer you found in Step 1**

is an improper fraction, change it to a mixed number.

Write down the fractioanl part, and carry the whole-number part

to the whole-number column.

3. **Add the whole-number parts**

(includin any number carried).

For example, suppose you're trying to add $8\frac{3}{5}+6\frac{4}{5}$.

Here's how to go about that:

1. **Add the fractions.**

$$\frac{3}{5}+\frac{4}{5}=\frac{7}{5}$$

2. **Switch improper fractions to mixed numbers,**

write down the fractional part,

and carry over the whole number.

Because the sum is an improper fraction,

convert it to the mixed number $1\frac{2}{5}$.

Write down $\frac{2}{5}$

and carry the 1 over to the whole-number column.

3. **Add the whole-number parts,**

including any whole numbers you carried over

when you converted to a mixed number.

1 + 8 + 6 = 15

Here's how the solved problem looks in column form.

(Be sure to line up the whole numbers in one column and the fractions in another.)

$$\begin{array}{r} \overset{1}{8}\frac{3}{5} \\ +6\frac{4}{5} \\ \hline 15\frac{2}{5} \end{array}$$

You will see the most difficult type of mixed-number addition

when the denominators of the fraction are different.

This difference doesn't change Steps 2 or 3,

but it does make Step 1 a little more challenging.

For example, suppose you want to add

$16\frac{3}{5}$ and $7\frac{7}{9}$.

1. **Add the Fractions.**

Add

$\frac{3}{5}$ and $\frac{7}{9}$,

As I show you earlier in "Adding Fractions with Different Denominators,"

you get the numerator (top number) of the answer

by cross-multiplying the two fractions and adding the results
(3 X 9 + 7 X 5);

you get the new denominator (bottom number)

by multiplying the two denominators (5 X 9):

$$\frac{3}{5}+\frac{7}{9}=\frac{3\cdot 9+7\cdot 5}{5\cdot 9}=\frac{27+35}{45}=\frac{62}{45}$$

2. **Convert improper fractions to mixed numbers,**

write down the fractional part,

and carry over the whole number.

The fraction $\frac{62}{45}$ is improper,

so change it to the mixed number $1\frac{17}{45}$.

Fortunately, the fractional part of this mixed number

is not reducible.

Write down the $\frac{17}{45}$

and carry over the 1 to the whole-number column.

3. **Add the whole numbers.**

1 + 16 + 7 = 24

Here's how the completed problem looks:

$$\begin{array}{r} \overset{1}{1}6\frac{3}{5} \\ +\ 7\frac{7}{9} \\ \hline 24\frac{17}{45} \end{array}$$

Subtracting Mixed Numbers

The basic way to subtract mixed numbers

is pretty close to the way you add them.

As previously stated,

the subtraction looks more like what you're used to with whole numbers.

REMEMBER:

Here's how to subtract two mixed numbers:

1. **Find the difference of the fractional parts.**

2. **Find the difference of the two whole-number parts.**

As with addition,

the subtraction is a lot easier

when the denominators are the same.

Borrowing with Mixed Numbers

One thing that makes things more complicated

is when you try to subtract a larger fractional part

from a smaller one.

As an example,

suppose you'd like to find

$11\frac{1}{6} - 2\frac{5}{6}$.

If you try subtracting the problems,

you get

$$\frac{1}{6}-\frac{5}{6}=-\frac{4}{6}$$

Obivously,

you don't want to have a negative number in your answer.

You can handle this problem

by borrowing from the column to the left.

This idea is quite similar to the borrowing

that you use in regular subtraction,

with a key difference.

TIP:

When borrowing in mixed-number subtraction,

do the following:

1. **Borrow 1 from the whole-number portion**

and add it to the fractional part.

This will turn the fraction into a mixed number

To find $11\frac{1}{6}-2\frac{5}{6}$,

borrow 1 from the 11 and add it to

$1\frac{1}{6}$:

$$11\frac{1}{6}=10+1\frac{1}{6}$$

2. **Convert this new mixed number into an improper fraction.**

Here's what you get when you change $1\frac{1}{6}$

to an improper fraction:

$$10 + 1\frac{1}{6} = 10\frac{7}{6}$$

The result is $10\frac{7}{6}$.

This answer is a weird cross

between a mixed number and an improper fraction,

but it's exactly what you need to handle the job!

3. **Use the result in your subtraction.**

$$\begin{array}{r} 10\frac{7}{6} \\ -\ 2\frac{5}{6} \\ \hline 8\frac{2}{6} \end{array}$$

In this particular case,

you need to reduce the fractional part of the answer:

$$8\frac{2}{6} = 8\frac{1}{3}$$

Unequal Denominators: Checking to See if You Need to Borrow

Suppose you want to subtract $15\frac{4}{11} - 12\frac{3}{7}$.

Because the denominators are different,

subtraction becomes more of a challenge.

But, you have another question to think about:

In this problem, do you need to borrow?

If $\frac{4}{11}$ is greater than $\frac{3}{7}$,

you don't have to borrow.

But if $\frac{4}{11}$ is less than $\frac{3}{7}$, you do.

TIP:

Here's how to test two fractions to see which is greater

by cross-multiplying:

$$\frac{3}{7}$$

$$3 \cdot 11 = 33$$

Because 28 is less than 33, $\frac{4}{11}$

is less than $\frac{3}{7}$.

You do have to borrow!

I get borrowing out of the way first:

$$15\frac{4}{11} = 14 + 1\frac{4}{11} = 14\frac{15}{11}$$

Now, the problem looks like this:

$$14\frac{15}{11} - 12\frac{3}{7}$$

The first step-subtracting the fractions-

is going to be the most time-consuming.

AS I show you earlier in "Subtracting Fractions with Different Denominators,"

you can go ahead and take care of that:

$$\frac{15}{11} - \frac{3}{7} = \frac{15 \cdot 7 - 3 \cdot 11}{11 \cdot 7} = \frac{105 - 33}{77} = \frac{72}{77}$$

The good news is that this fraction can't be reduced any further!

(It can't be reduced because 72 and 77 have no common factors:

72 = 2 X 2 X 2 X 3 X 3, and 77 = 7 X 11.)

The hard part of the problem is done,

and the rest flows quite easily:

$$\begin{array}{r} 14\frac{15}{11} \\ -\ 12\frac{3}{7} \\ \hline 2\frac{72}{77} \end{array}$$

This problem is as difficult as a mixed-number subtraction problem gets.

Look over it step-by-step.

Better yet,

copy the problem, close this book,

and try to solve the problem.

If you get stuck, it's okay.

Open the book back up!

I'll be right here with you!